(GO1-2) Determination of Refractive Index of a Transparent Liquid Using a Concave Mirror

Aim of experiment

Determination of refractive index of a transparent liquid by using a concave mirror

Apparatus

Concave Mirror- Support With a Vertical Rod, and a Clamp Holding an Arrow Like Light Weight Thin Aluminum Sheet- Meter Ruler.

Theory of experiment

Measuring the refractive index n of a substance or medium is part of various approaches to determine this index have been developed over the years based on the different ways light reflects and transmits in the medium.

Among all those different methods, the spherical concave mirror filled with a liquid makes itself an excellent alternative to measure the refractive index of the liquid, specially when no advanced apparatuses are available, and the accuracy of the measurements is not critical.

A simple geometrical derivation of the refractive index of a transparent liquid that is obtained using a spherical concave mirror will be presented.

This derivation relies mostly on Snell's law and the small-angle approximation. This method is based on the measurements of the actual and apparent position of the centre of curvature of the mirror, when it is empty and filled with a liquid, respectively.

Let us first consider the spherical concave mirror shown in *figure 1.* The optical axis is the radial line through the centre of the mirror that intersects its surface at the vertex point V.

Some relevant points on the optical axis are the centre of curvature C, and the focal point F. The centre of curvature coincides with the centre of the sphere of which the mirror forms a section. At the focal point, rays parallel to the optical axis and incident on the concave mirror, intersect after being reflected by the mirror's surface.

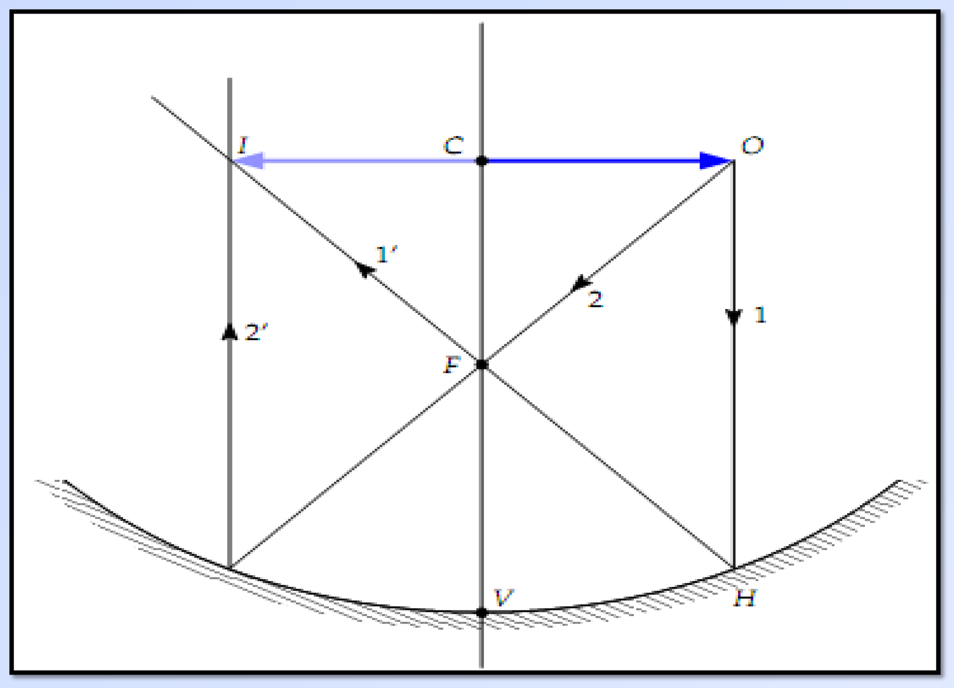

Figure 1

Figure 1. Positions of the focal point and centre of curvature of a spherical concave mirror. Rays 1 and 2, and their respective reflections 1′ and 2′ in the mirror, determine the location of the image.

The distance CV between the centre of curvature and the vertex is equal to the radius of the sphere and is called *the radius of curvature R*. Similarly, the distance FV between the focal point of the mirror and the vertex is known as *the focal length f*. In general, when the rays are close to the optical axis that is, for the small-angle approximation, the focal length can be shown to be half of the radius of curvature:

$$f = R/2 \qquad (1)$$

The location and nature of the image formed by a spherical mirror can be determined by *graphical ray-tracing techniques* .

To find the conjugate image point I of an object point O located at the centre of curvature C, the paths of any two rays leaving O are sufficient.

We first use the so-called parallel ray 1 that is incident along a path parallel to the optical axis, strikes the mirror at point H, and is reflected through the focal point F as ray 1′. Next, we use the so-called focal ray 2 that passes through the focal point and is reflected parallel to the optical axis as ray 2′. The image point I is formed where the two rays 1′ and 2′ intersect. This image is real, inverted, located at the centre of curvature C, and has the same size of the object, as shown in figure 1

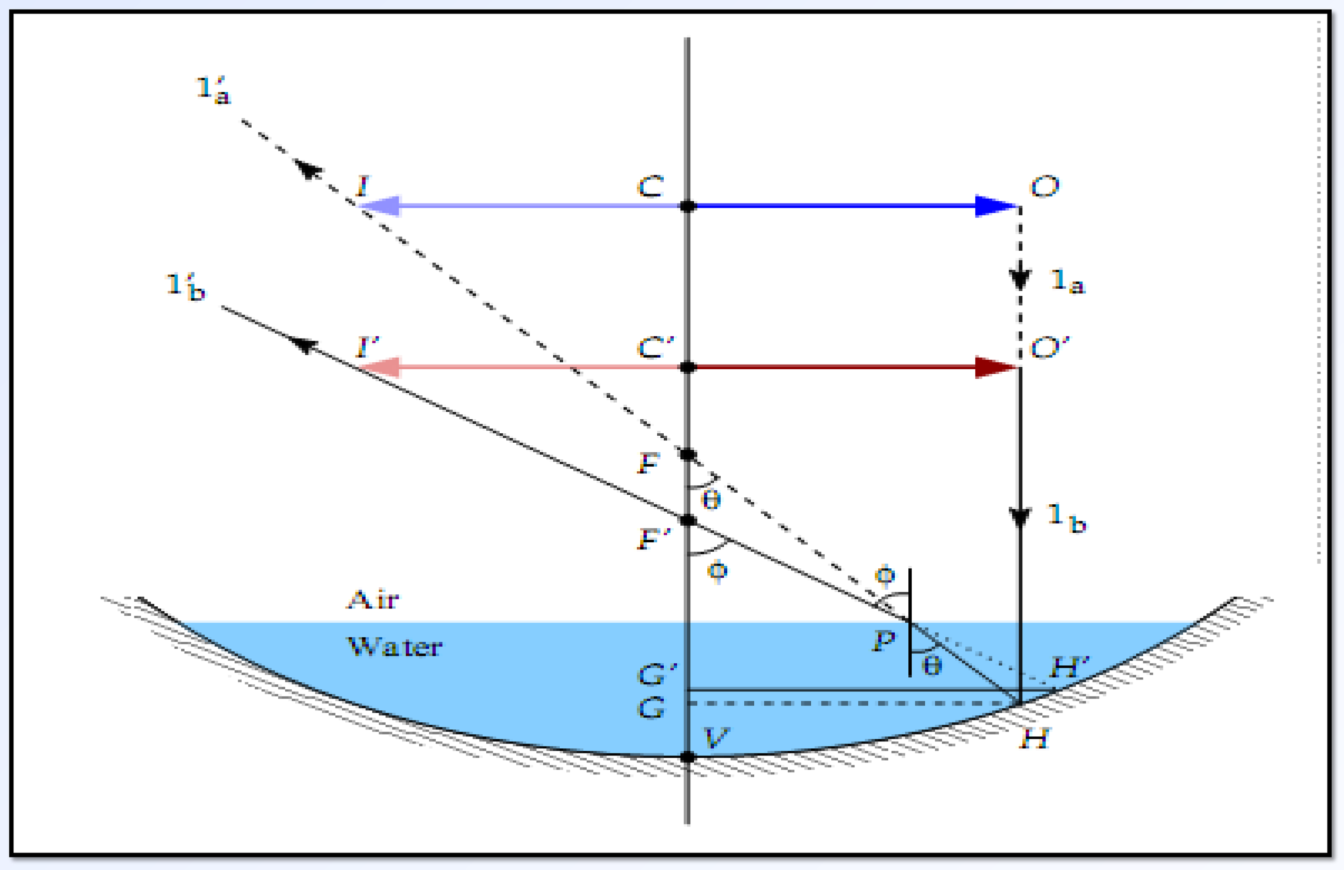

Figure 2. Formation of the images at the positions of the actual and apparent centres of curvature.

When the point object O is placed at the centre of curvature C, and the mirror is empty (no water has been poured in), we may use the same graphical ray-tracing methods of figure 1 to locate the conjugate image point I. Ray 1a leaves point O parallel to the optical axis, strikes the mirror at point H and reflects as ray 1′. This ray intersects the principal axis at focal point F. The focal length is then

$$f = FV .$$

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The conjugate image point I is formed at the centre of curvature C, as illustrated in figure 2. The image is real, inverted and has the same size of the object.

Now, when the mirror is filled with a thin layer of water, the magnitude of its focal length decreases, and the apparent centre of curvature C′ gets closer to the vertex V of the mirror. Ray 1a leaves object point O parallel to the optical axis, strikes the mirror at point H and then reflects. The reflected ray refracts from water into air bending away from the surface normal at the point of incidence P, for the refractive index of water n_w is bigger than that of air n_a. The refracted ray 1′b intersects the optical axis at the new focal point F′. The new focal length becomes

$$f' = F'V \,.$$

If the object is placed at the apparent centre of curvature C′, and labeled object point O′, the conjugate image point I′ is found at the apparent centre of curvature C′. The image is real, inverted, and has the same size of the object (see figure 2).

Using Snell's law at surface point P, we get

$$n_w \sin\theta = n_a \sin\varphi, \qquad (2)$$

where n_w and n_a are the refractive indices of water and air, respectively. From this point on, we will assume $n_a \approx 1.00$.

Figure 2 shows that from triangles FHG and F′H′G′

$$HG = FH \sin\theta \qquad (3a)$$
$$H'G' = F'H' \sin\varphi \qquad (3b)$$

The backward extension of the refracted ray 1′b strikes the mirror at point H′

If we assume that the focal lengths F and F′ are large compared to the thickness of the water layer in the mirror, points G and G′, as well as, H and H′ are very close to each other. Therefore, we can make the following approximations for the geometrical lengths

$$H'G' \approx HG \qquad (4a)$$
$$F'H' \approx F'H. \qquad (4b)$$

Physically, points H and H′ , as well as G and G′ are not coinciding but are located nearby. Using (5) in (4b), we may write

$$HG = F'H \sin\varphi \qquad (5)$$

Since the left hand sides of (4a) and (4b) are the same, we may equate both equations to obtain

$$FH \sin\theta = F'H \sin\varphi. \qquad (6)$$

Using (3) in (7), we obtain

$$FH/F'H = \sin\varphi/\sin\theta = n_w. \qquad (7)$$

If the angles θ and φ are small, then these angles can be replaced by their tangents (small-angle approximation). Furthermore, the distance GV in figure 2, *the sagittal depth of the surface*, is also small, and we may neglect it. Therefore, we can write

$$n_w = \tan\varphi/\tan\theta = f/f' \qquad (8)$$

Finally, using (1), we can express the refractive index of water in terms of the actual and apparent radii of curvature as follows

$$n_w = R/R' \qquad (9)$$

Procedure

1. The clamp that holds the arrow is moved to position C until a sharp image of the arrow is formed by the empty mirror. The radius of curvature R is then measured.
2. The mirror is now filled with a thin layer of water, and the arrow and screen moved down until a new sharp image of the lamp is formed. This position corresponds to the apparent centre of curvature C′. The new radius of curvature R′ is then measured.
3. The refractive index can be obtained by using the equation

$$n_w = R/R'$$

4. Repeat steps 1 – 3 two extra times and tabulate your results

Results

Trial	R cm	R' cm	$n_w = R/R'$
1			
2			
3			
		n_{wav}=	